problem is obtained from the weak inclusion theorem. Then a strong inclusion theorem is derived in the case of linear optimization.

10. Locher, F.: "Optimale definite Polynome und Quadraturformeln [Optimal Definite Polynomials and Quadrature Formulas]."

A definite polynomial is defined as one which does not change sign on $[-1,+1]$. The problem considered is that of finding the extreme value for $\int_{-1}^{+1} p_n(x)w(x)dx$ with p_n taken from among all definite monic polynomials of degree $\leqq n$ and $w(x)$ a given nonnegative weight function. Application of the solution to this problem is made to determine some quadrature formulas and to solve a problem posed by P. Kirchberger.

11. Sibony, M.: "Some Numerical Techniques for Optimal Control Governed by Partial Differential Equation" (sic).

Considered as problem (1) is: find $u \in X$ so that $F(u) \leqq F(v)$ for all $v \in X$, where X is a closed convex subset of a Banach space and F is a given functional. A reformulation of this with greater detail in the case of control problems governed by partial differential equations is given as problem (2). Several illustrations of obtaining numerical solutions for both problem types are presented.

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32 [2.05.3, 2.20, 2.40].—RICHARD P. BRENT, *Algorithms for Minimization Without Derivatives*, Prentice-Hall, Englewood Cliffs, N. J., 1973, xii + 195 pp., 24 cm. Price \$12.—.

This well written very readable book should be of particular interest to numerical analysts working on methods for finding zeros and extrema of functions. The book is concerned primarily with functions of a single variable and exclusively with methods which use only function values; no derivative evaluations are required by any of the algorithms presented. It also emphasizes algorithmic details that are extremely important for developing reliable computer codes.

In the first chapter, a very useful summary is given of the material covered in subsequent chapters. As these chapters are relatively self-contained, this enables the reader to easily determine which sections of the book to read according to his interests.

Fundamental results on Taylor series, polynomial interpolation, and divided differences are found in Chapter 2. These results are proved under slightly weaker assumptions than one usually finds in the literature. Chapter 3 presents a unified treatment of successive polynomial interpolation for finding zeros of a function and its derivatives; i.e., zeros, stationary points, inflexion points, etc.

The next four chapters are devoted, respectively, to algorithms for:

1. Finding a zero of a function, given an interval in which it changes sign.
2. Finding a local minimum of a function defined on a given interval.
3. Finding the global minimum of a function of one or several variables, given upper bounds on the second derivatives. (This is practical for at most two or three variables.)
4. Finding a local minimum of a function of several variables.

In each case, convergence, the rate of convergence and the effect of rounding errors are analyzed. ALGOL programs that are claimed to be fast and reliable in the presence of roundoff are given along with a substantial amount of numerical results which supports these claims. FORTRAN implementations of the first two algorithms are also given in the Appendix. The book also contains an extensive and up-to-date bibliography relevant to nonlinear optimization.

The reviewer's only complaint about the book is its title which implies that it is about the minimization of nonlinear functions of several variables. Actually, less than one-third of the book deals with this subject. The title, "Algorithms for finding zeros and extrema of functions without calculating derivatives," of an earlier version of the book that appeared as a Stanford University report, gives a more accurate description of the book's contents.

In any case, this book is an excellent reference work for optimizers and root finders who should find the programs in it of great value.

D. G.

33 [2.45, 10].—W. W. Boone, F. B. Cannonito & C. R. Lyndon, Editors, *Word Problems. Decision Problems and the Burnside Problem in Group Theory,* North-Holland Publishing Co., Amsterdam and London, 1973, viii + 646 pp., 23 cm. Price \$42.—.

This work is a collection of papers which were presented at a conference at the University of California, Irvine, in September, 1969. A detailed review of all of the 21 papers would require an excessive amount of space; the potential reader will have to consult the "Mathematical Reviews" for an item by item review. However, the book has a unifying theme, namely, the use of algorithmic methods, and this fact is well presented in a very good introduction by the editors who also give proper credit to the pioneering work of Thue and Dehn.

Of the 646 pages of the book, 282 are occupied by a paper on "The existence of infinite Burnside groups" by J. J. Britton. It contains a new proof of the Novikov-Adjan Theorem that, for all sufficiently large odd numbers e, the Burnside groups with exponent e and at least two generators are infinite. The paper is practically self-contained, using little more than the concepts of free group and free product (according to a claim made by the author). Nevertheless, it is probably one of the most difficult mathematical papers ever to appear in print. It is impossible to skim it, and it is very important